Field Goal for the Win

Student Activity

Name ____________________

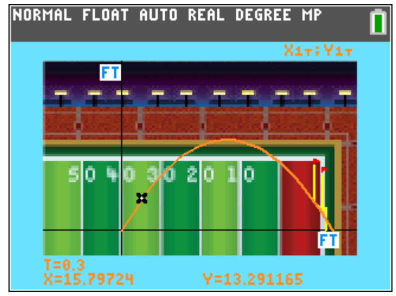
Class ____________________

Get Your Game On

Four seconds left in the Big Game and the score is 27 to 28 - your team is behind. A 50-yard field goal wins the game. Your task is to use a mathematical model to demonstrate kicking a field goal to win the game.

Press [prgm] and run the program THEKICK.

1. The Kick



Follow the on-screen prompts to the main menu.
Press 1 to see the 50-yard kick. Then press [trace].

a. Investigate the information on the screen. Notice the three variables, T, X, and Y. Press the right arrow three times and record the values of the variables to the nearest thousandth.

T=________________________________

X=________________________________

Y=________________________________

b. Interpret the values of these variables in context, including units.

c. Trace on the function to a different point and stop. Record T, X, and Y, then discuss what these values mean with your group.

d. Using the Trace feature, graphically investigate the time when the ball is 150 feet from where it was kicked in the horizontal direction. Between which two values of T does the ball pass 150 feet downfield? Record your answers to the nearest tenth of a second. How high is the ball at each of these times?

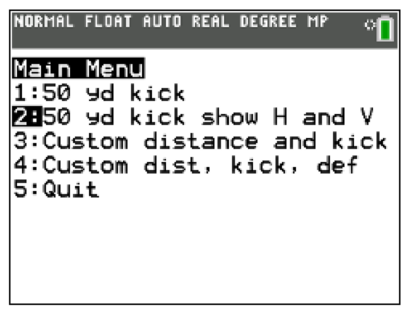
1. The Kick (continued)

e. Using the trace feature, graphically investigate a time when the ball is 10 feet high. Between which two values of T does the ball reach 10 feet high? Record your answers to the nearest tenth of a second. How far downfield is the ball at each of these times?

f. Does it appear that you made the field goal? How can you tell? Can you prove you made the field goal graphically? Justify your response.

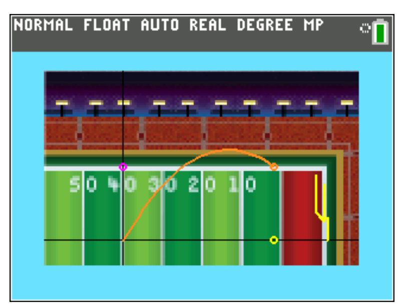
2. Modeling Horizontal Motion

In order to answer question 1f. algebraically instead of on the graph, we need a function to model the horizontal distance the ball travels downfield after it's kicked as a function of time, $x(t)$.



Run the program THEKICK.
On the main menu, press 2 to see a kick with the horizontal and vertical components shown. The angle of the kick is 43 degrees and the velocity of the kick is 72 ft/s. Press [enter] and watch the kick. You will be placed into trace mode. Press [enter] when you are done exploring.

a. Repeat the kick and watch the horizontal component (yellow). Does its speed increase, decrease, or remain constant? What does that tell you about the kind of equation that will model the horizontal motion?

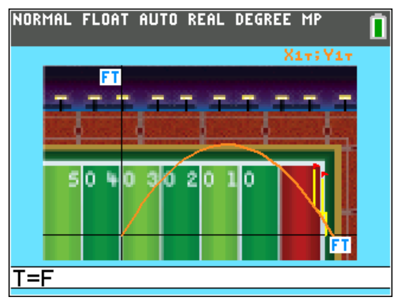


2. Modeling Horizontal Motion (continued)

b. The function, for the horizontal distance traveled downfield, if the ball is kicked with initial velocity 72 ft/s and an angle of 43 degrees is $x(t) = 72 \cdot \cos(43°) \cdot t$. Is $x(t)$ linear or nonlinear?

c. How long will it take for the ball to travel 150 feet? Exit the program, then solve $x(t) = 150$ and store your answer in variable F.

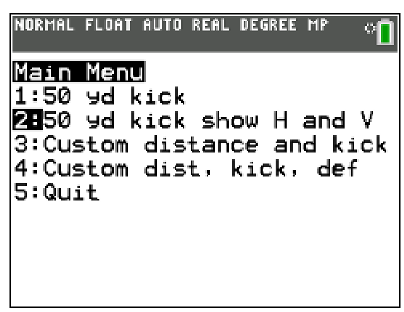
d. Press [trace]. Using the Trace feature, trace to the value for F and explain what the numbers on the screen mean.



e. Did you make the kick and win the game? How do you know?

3. Modeling Vertical Motion

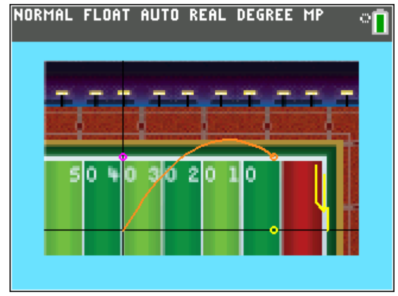
A second way to answer question 1f. algebraically instead of graphically, uses a function to model the height of the ball during a kick as a function of time, y(t).



Run the program THEKICK.
On the main menu press 2 to see a kick with the horizontal and vertical components shown. The angle of the kick is 43 degrees and the velocity of the kick is 72 ft/s. Press [enter] and watch the kick. You will be placed into Trace mode. Press [enter] when you are done exploring.

3. Modeling Vertical Motion (continued)

a. Repeat the kick (Option 1 in the submenu) and this time, pay attention to the vertical component (light pink). Describe its motion. Include its speed and direction in your description. Does its speed increase, decrease, remain constant? What does that tell you about the kind of equation that will model the vertical component?

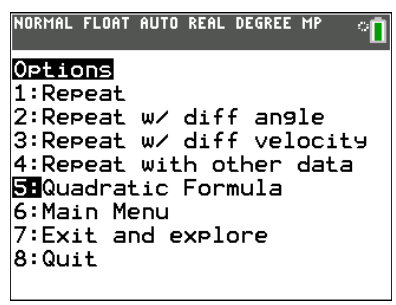


b. What other factors besides the kick contribute to the vertical position of the ball after it's kicked?

Acceleration due to gravity is -32 ft/s^2. The negative is because the ball is being pulled down toward the ground. When modeling vertical position, we can apply physics laws to know we can add the initial velocity multiplied by the time (this gives how high the ball would be if there were no gravity) and $-\frac{1}{2}g \cdot t^2$. The function for modeling the ball's height, $y(t)$, if it is kicked with an initial velocity of 72 ft/s at an angle of 43 degrees is: $y(t) = 72 \cdot \sin(43°) \cdot t - 16 \cdot t^2$.

c. What kind equation is $y(t)$? Is it linear or nonlinear?

d. Solve for $y(t) = 10$ to determine when the ball is 10 feet in the air. (Hint: You may want to use the quadratic formula Option in the program. It is Option 5 in the submenu after you've drawn the graph.) Use the Trace feature to confirm your solution.



e. Explain how your solution is consistent to your answer from 2e.

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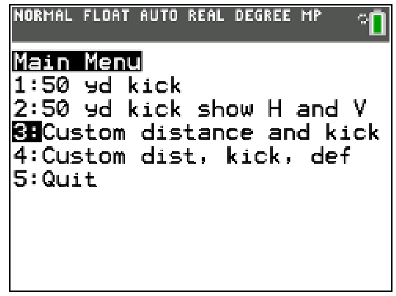
Extensions:

4. **<u>Application of the model</u>**



Run the program Field Goal again and select Option 3. You're going to kick a field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle, and run the program to graph your kick.

Length of kick (yards): ______ Angle (degrees): ___________
Velocity (ft/s): _____________

The equations you used in Questions 2 and 3 to model the horizontal and vertical positions of the ball can be used at the same time to model the actual flight of the ball after it is kicked.

a. Based on the graph, can you tell if the ball passes above the 10-foot crossbar on the goal posts? How can you tell?

b. Fill in the equation for $x(t)$ and solve algebraically for when the ball passes through the goal posts. Use your solution to decide if the field goal is made or not.

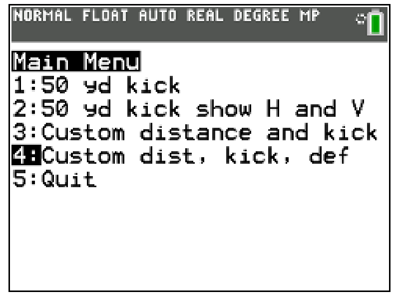
$x(t) = \_\_\_\_ \cos(\_\_\_) \cdot t$

c. Attempt the kick with a different angle or velocity, and determine graphically if the kick is made using the program. Then use algebra to confirm your answer.

Angle ________ Velocity _____________ Made (Y/N) _________

5. **<u>But what about the defense?</u>**

Run the program THEKICK again and select Option 4. You're going to kick a field goal to win the game. Professional kickers kick the ball with a velocity of about 70 to 88 ft/s (48 to 60 mph) and at an angle that varies between 27 and 43 degrees. Choose your velocity and kick angle, and run the program to graph your kick. The kicker kicks from 7 yards behind the line of scrimmage, the defense typically gets little or no rush (between 0 and 2 yards) and defensive players can reach about 8 to 9 feet in the air.



Length:__________ Angle: _________ Velocity: ___________

Rush: __________ Reach: __________

a. Based on your model, will a defender block the kick? Defend your answer graphically and algebraically.

b. Based on your model, will the ball pass above the 10-foot crossbar on the goal posts? How can you tell? Defend your answer more than one way.

c. Attempt the kick with a different angle or velocity and determine algebraically if the kick makes it over the defense and is good. Then use the program to confirm your answer.

Angle _________ Velocity ___________ Rush _________ Reach _______

Blocked (Y/N) _________ Made (Y/N) _____________